

## Answers to Selected Exercises

### 5. Special Distributions

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#### 1. Introduction

- 1.6.  $f(x) = \frac{1}{b} \exp\left(-\frac{x-a}{b}\right), x > a$
- 1.7.  $f(x) = \frac{1}{\pi b \left(1 + \frac{x-a}{b}\right)^2}, x \in \mathbb{R}$

#### 2. The Normal Distribution

- 2.22. Let  $X$  denote the volume of beer in liters
- a.  $\mathbb{P}(X > 0.48) = 0.9772$
  - b.  $x_{0.95} = 0.51645$
- 2.23. Let  $X$  denote the radius of the rod and  $Y$  the radius of the hole.  $\mathbb{P}(Y - X < 0) = 0.0028$ .
- 2.24. Let  $X$  denote the combined weight of the 5 peaches, in ounces.  $\mathbb{P}(X > 45) = 0.0127$ .

#### 3. The Gamma Distribution

- 3.8.  $\mathbb{P}(X > 3) = \frac{17}{2} e^{-3} \approx 0.432$
- 3.9.

- a.  $Q_1 = 0.287, Q_2 = 0.693, Q_3 = 1.396, Q_3 - Q_1 = 1.109,$
- b.  $Q_1 = 0.961, Q_2 = 1.678, Q_3 = 2.692, Q_3 - Q_1 = 1.731,$
- c.  $Q_1 = 1.727, Q_2 = 2.674, Q_3 = 3.920, Q_3 - Q_1 = 2.193,$

☑ 3.14. Let  $X$  denote the petal length in centimeters.

- a.  $\mathbb{E}(X) = 4$
- b.  $\text{sd}(X) = 2$

☑ 3.21. Let  $X$  denote the lifetime in hours.

- a.  $\mathbb{P}(X > 300) = 13e^{-3} \approx 0.6472$
- b.  $\mathbb{E}(X) = 400$
- c.  $\text{sd}(X) = 200$

☑ 3.26.

- a.  $\mathbb{P}(18 < X < 25) = 0.3860, \mathbb{P}(18 < X < 25) \approx 0.4095$
- b.  $y_{80} = 25.038, y_{80} \approx 25.325$

#### 4. The Chi-Square Distribution

☑ 4.5.

- a.  $Q_1 = 0.102, Q_2 = 0.455, Q_3 = 1.323, Q_3 - Q_1 = 1.221,$
- b.  $Q_1 = 0.575, Q_2 = 1.386, Q_3 = 2.773, Q_3 - Q_1 = 2.198,$
- c.  $Q_1 = 2.675, Q_2 = 4.351, Q_3 = 6.626, Q_3 - Q_1 = 3.951,$
- d.  $Q_1 = 6.737, Q_2 = 9.342, Q_3 = 12.549, Q_3 - Q_1 = 5.812,$

☑ 4.14. Let  $Z$  denote the distance from the missile to the target.  $\mathbb{P}(Z < 20) = 1 - e^{-2} \approx 0.8647.$

☑ 4.16.

- a.  $\mathbb{P}(15 < X < 20) = 0.3252, \mathbb{P}(15 < X < 20) \approx 0.3221$
- b.  $x_{0.75} = 21.605, x_{0.75} \approx 22.044$

#### 5. The Student $t$ Distribution

☑ 5.6.

- a.  $Q_1 = -1, Q_2 = 0, Q_3 = 1, Q_3 - Q_1 = 2,$
- b.  $Q_1 = -0.816, Q_2 = 0, Q_3 = 0.816, Q_3 - Q_1 = 1.632,$
- c.  $Q_1 = -0.727, Q_2 = 0, Q_3 = 0.727, Q_3 - Q_1 = 1.454,$
- d.  $Q_1 = -0.7, Q_2 = 0, Q_3 = 0.7, Q_3 - Q_1 = 1.4,$

☑ 5.13.

- a.  $\mathbb{P}(-0.8 < T < 1.2) = 0.650$ ,  $\mathbb{P}(-0.8 < T < 1.2) \approx 0.673$   
 b.  $x_{0.90} = 1.372$ ,  $x_{0.90} \approx 1.281$

## 6. The $F$ distribution

☑ 6.4.

- a.  $Q_1 = 0.528$ ,  $Q_2 = 1$ ,  $Q_3 = 1.895$ ,  $Q_3 - Q_1 = 1.367$ ,  
 b.  $Q_1 = 0.529$ ,  $Q_2 = 932$ ,  $Q_3 = 1.585$ ,  $Q_3 - Q_1 = 1.056$ ,  
 c.  $Q_1 = 0.631$ ,  $Q_2 = 1.073$ ,  $Q_3 = 1.890$ ,  $Q_3 - Q_1 = 1.259$ ,  
 d.  $Q_1 = 0.645$ ,  $Q_2 = 1$ ,  $Q_3 = 1.551$ ,  $Q_3 - Q_1 = 0.906$ ,

## 8. The Beta Distribution

☑ 8.14.

- a.  $Q_1 = 0.25$ ,  $Q_2 = 0.5$ ,  $Q_3 = 0.75$ ,  $Q_3 - Q_1 = 0.5$ ,  
 b.  $Q_1 = 0.091$ ,  $Q_2 = 0.206$ ,  $Q_3 = 0.370$ ,  $Q_3 - Q_1 = 0.279$ ,  
 c.  $Q_1 = 0.630$ ,  $Q_2 = 0.794$ ,  $Q_3 = 0.9009$ ,  $Q_3 - Q_1 = 0.279$ ,  
 d.  $Q_1 = 0.194$ ,  $Q_2 = 0.314$ ,  $Q_3 = 0.454$ ,  $Q_3 - Q_1 = 0.260$ ,  
 e.  $Q_1 = 0.546$ ,  $Q_2 = 0.686$ ,  $Q_3 = 0.806$ ,  $Q_3 - Q_1 = 0.260$ ,  
 f.  $Q_1 = 0.379$ ,  $Q_2 = 0.5$ ,  $Q_3 = 0.621$ ,  $Q_3 - Q_1 = 0.242$ ,

## 9. The Weibull Distribution

☑ 9.7.  $Q_1 = 0.5364$ ,  $Q_2 = 0.8326$ ,  $Q_3 = 1.1774$ ,  $Q_3 - Q_1 = 0.6411$ ,

☑ 9.24.

- a.  $\mathbb{P}(T > 1500) = 0.1966$   
 b.  $\mathbb{E}(T) = 940.656$ ,  $\text{sd}(T) = 787.237$   
 c.  $h(t) = 0.000301 t^{0.2}$

## 10. The Zeta Distribution

☑ 10.3.  $\mathbb{P}(X > 4) = 1 - \frac{49}{6\pi^2} \approx 0.1725$

☑ 10.7.

- a.  $\mathbb{E}(X) = 1.1106$   
 b.  $\text{sd}(X) = 0.5351$

## 11. The Pareto Distribution

☑ 11.6.  $Q_1 = 1.1006$ ,  $Q_2 = 1.2599$ ,  $Q_3 = 1.5874$ ,  $Q_3 - Q_1 = 0.4868$

☑ 11.16. Let  $X$  denote income.

a.  $\mathbb{P}(2000 < X < 4000) = 0.1637$  so the proportion is 16.37%

b.  $Q_2 = 1259.92$

c.  $Q_1 = 1100.64$ ,  $Q_3 = 1587.40$ ,  $Q_3 - Q_1 = 486.76$

d.  $\mathbb{E}(X) = 1500$

e.  $\text{sd}(X) = 866.03$

f.  $F^{-1}(0.9) = 2154.43$

## 12. The Logistic Distribution

☑ 12.2.  $\mathbb{P}(-1 < X < 2) = 0.6119$

☑ 12.7.  $Q_1 = -1.0986$ ,  $Q_2 = 0$ ,  $Q_3 = 1.0986$ ,  $Q_3 - Q_1 = 2.1972$

☑ 12.8.  $F^{-1}(0.1) = -2.1972$ ,  $F^{-1}(0.9) = 2.1972$

## 13. The Lognormal Distribution

☑ 13.6.  $\mathbb{P}(X > 20) = 0.1497$

☑ 13.7.  $Q_1 = 0.5097$ ,  $Q_2 = 1$ ,  $Q_3 = 1.9621$ ,  $Q_3 - Q_1 = 1.4524$

☑ 13.11.

a.  $\mathbb{E}(X) = e^{\frac{5}{2}} \approx 12.1825$

b.  $\text{sd}(X) = \sqrt{e^6 - e^5} \approx 15.9629$

## 15. Benford's Law

☑ 15.4.

a.  $f(y) = \frac{1}{0.2303 y}$  for  $y \in [\frac{1}{10}, 1)$ .

b.  $\mathbb{E}(Y) = 0.3909$ ,  $\text{var}(Y) = 0.0622$ ,

☑ 15.6.

a.

$n$	$\mathbb{P}(N_1 = n)$
1	0.3010
2	0.1761
3	0.1249

4	0.0969
5	0.0792
6	0.0669
7	0.0580
8	0.0512
9	0.0458

b.  $E(N_1) = 3.4402$ ,  $\text{var}(N_1) = 6.0567$

15.8.

$$\mathbb{P}(N_1 = n_1, N_2 = n_2)$$

$n_2 \setminus n_1$	1	2	3	4	5	6	7	8	9
0	0.0414	0.0212	0.0142	0.0107	0.0086	0.0072	0.0062	0.0054	0.0048
1	0.0378	0.0202	0.0138	0.0105	0.0084	0.0071	0.0061	0.0053	0.0047
2	0.0348	0.0193	0.0134	0.0102	0.0083	0.0069	0.0060	0.0053	0.0047
3	0.0322	0.0185	0.0130	0.0100	0.0081	0.0068	0.0059	0.0052	0.0046
4	0.0300	0.0177	0.0126	0.0098	0.0080	0.0067	0.0058	0.0051	0.0046
5	0.0280	0.0170	0.0122	0.0092	0.0078	0.0066	0.0058	0.0051	0.0045
6	0.0263	0.0164	0.0119	0.0093	0.0077	0.0065	0.0057	0.0050	0.0045
7	0.0248	0.0158	0.0116	0.0091	0.0076	0.0064	0.0056	0.0050	0.0045
8	0.0235	0.0152	0.0113	0.0090	0.0074	0.0063	0.0055	0.0049	0.0044
9	0.0223	0.0147	0.0110	0.0088	0.0073	0.0062	0.0055	0.0049	0.0044

15.10.

a.

$n$	$\mathbb{P}(N_2 = n)$
0	0.1197
1	0.1139
2	0.1088
3	0.1043
4	0.1003
5	0.0967
6	0.0934
7	0.0904
8	0.0876

9	0.0850
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b.  $\mathbb{E}(N_2) = 4.1874$ ,  $\text{var}(N_2) = 0.8254$

15.12.

a. 0.0008171

b. 0.001377

c. 0.003205

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