

Answers to Selected Exercises

6. Random Samples

3. The Sample Variance
 4. Partial Sums and the Central Limit Theorem
 6. Order Statistics
 7. Sample Covariance and Correlation
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3. The Sample Variance

- 3.22. $M = 852.4, S = 79.0$
- 3.23. $M = 5.45, S = 0.22$
- 3.24. $M = 49.215, S = 1.522$
- 3.25.
 - a. $M = 37.787, S = 17.776$
 - b. $M = 14.62, S = 1.737$
 - c. $M = 55.52, S = 30.459$
 - d. $M = 43.22, S = 28.747$

4. Partial Sums and the Central Limit Theorem

- 4.18.
 - a. 0.8682
 - b. 17.03
- 4.19.
 - a. 0.2071
 - b. 1.531
- 4.21. 0.6741
- 4.24.
 - a. 0.3063
 - b. 25.32
- 4.26.
 - a. 0.4107
 - b. 24.3

☑ 4.27.

a. 0.5448

☑ 4.28. 0.5383

☑ 4.29.

a. 0.6310

b. 0.6259

☑ 4.32.

a. 0.6318

b. 30.1

6. Order Statistics

☑ 6.9.

$$g_k(t) = \binom{n}{k-1, 1, n-k} r (1 - e^{-rt})^{k-1} e^{-r(n-k+1)t}, \quad t \geq 0$$

☑ 6.11.

$$b. \mathbb{E}(X_{n,k}) = \frac{k}{n+1}, \quad \text{var}(X_{n,k}) = \frac{k(n-k+1)}{(n+1)^2(n+2)}$$

☑ 6.13.

$$a. g_1(1) = \frac{671}{1296}, g_1(2) = \frac{41}{144}, g_1(3) = \frac{175}{1296}, g_1(4) = \frac{65}{1296}, g_1(5) = \frac{4}{432}, g_1(6) = \frac{1}{1296}$$

$$b. g_2(1) = \frac{19}{144}, g_2(2) = \frac{119}{432}, g_2(3) = \frac{121}{432}, g_2(4) = \frac{29}{144}, g_2(5) = \frac{41}{432}, g_2(6) = \frac{7}{432}$$

$$c. g_3(1) = \frac{7}{432}, g_3(2) = \frac{41}{432}, g_3(3) = \frac{29}{144}, g_3(4) = \frac{121}{432}, g_3(5) = \frac{119}{432}, g_3(6) = \frac{19}{144}$$

$$d. g_4(1) = \frac{1}{1296}, g_4(2) = \frac{4}{432}, g_4(3) = \frac{65}{1296}, g_4(4) = \frac{175}{1296}, g_4(5) = \frac{41}{144}, g_4(6) = \frac{671}{1296}$$

☑ 6.17.

$$g(y_1, y_2, \dots, y_n) = n! r^n \exp(-\sum_{i=1}^n y_i), \quad 0 < y_1 < y_2 < \dots < y_n$$

☑ 6.19. The joint probability density function g is defined on

$$\{(y_1, y_2, y_3, y_4) : (y_i \in \{1, 2, 3, 4, 5, 6\}) \text{ and } (y_1 \leq y_2 \leq y_3 \leq y_4)\}$$

a. $g(y_1, y_2, y_3, y_4) = \frac{1}{1296}$ if the coordinates of (y_1, y_2, y_3, y_4) are all the same (there are 6 such vectors).

b. $g(y_1, y_2, y_3, y_4) = \frac{4}{1296}$ if there are two distinct coordinates in (y_1, y_2, y_3, y_4) , one value occurring 3 times and the other value once (there are 30 such vectors).

c. $g(y_1, y_2, y_3, y_4) = \frac{6}{1296}$ if there are two distinct coordinates in (y_1, y_2, y_3, y_4) , each value occurring 2 times (there are 15 such vectors).

d. $g(y_1, y_2, y_3, y_4) = \frac{12}{1296}$ if there are three distinct coordinates in (y_1, y_2, y_3, y_4) , one value occurring

twice and the other values once (there are 60 such vectors).

e. $g(y_1, y_2, y_3, y_4) = \frac{24}{1296}$ if the coordinates in (y_1, y_2, y_3, y_4) , are distinct (there are 15 such vectors).

☑ 6.21.

b. $\mathbb{E}(R) = \frac{n-1}{n+1}, \text{var}(R) = \frac{2(n-1)}{(n+1)^2(n+2)}$

c. $R \rightarrow 1$ as $n \rightarrow \infty$

☑ 6.22. R has probability density function h given by $h(0) = \frac{6}{1296}, h(1) = \frac{70}{1296}, h(2) = \frac{200}{1296}, h(3) = \frac{330}{1296},$
 $h(4) = \frac{388}{1296}, h(5) = \frac{302}{1296}$

☑ 6.26. $\min = 620, Q_1 = 805, Q_2 = 850, Q_3 = 895, \max = 1070$. The true value is 734.5.

☑ 6.27. $\min = 4.88, Q_1 = 5.30, Q_2 = 5.46, Q_3 = 5.61, \max = 5.85$. The true value is 5.517.

☑ 6.28. $\min = 46.22, Q_1 = 48.28, Q_2 = 49.07, Q_3 = 50.23, \max = 52.06$.

☑ 6.29.

a. $\min = 43, Q_1 = 51, Q_2 = 58, Q_3 = 64, \max = 79$

b. $\min = 43, Q_1 = 48, Q_2 = 50, Q_3 = 52, \max = 58$

c. $\min = 49, Q_1 = 62, Q_2 = 65, Q_3 = 69, \max = 79$

d. $\min = 49, Q_1 = 56, Q_2 = 59, Q_3 = 63, \max = 70$

7. Sample Covariance and Correlation

☑ 7.38. Correlation between petal length and petal width:

a. 0.9559

b. 0.3316

c. 0.3496

d. 0.6162

☑ 7.39. Correlation matrix for the vector (red, green, blue, orange, yellow, brown):

$$\begin{pmatrix} 1 & 0.1496 & -0.2814 & -0.0985 & -0.1104 & -0.3795 \\ 0.1496 & 1 & 0.2895 & -0.3307 & -0.4322 & -0.111 \\ -0.2814 & 0.2895 & 1 & -0.2349 & -0.4349 & -0.0452 \\ -0.0985 & -0.3307 & -0.2349 & 1 & -0.1058 & -0.058 \\ -0.1104 & -0.4322 & -0.4349 & -0.1058 & 1 & -0.4303 \\ -0.3795 & -0.111 & -0.0452 & -0.058 & -0.4303 & 1 \end{pmatrix}$$

☑ 7.40.

a. $\text{pw} = -3.45 + 0.41 \text{pl}$

c. Predicted value: $pw = 12.8$ when $pl = 40$

☑ 7.41.

a. $sw = -5.44 + 0.783 sl$

c. Predicted value: $sw = 30.24$ when $sl = 45$

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