Answers to Selected Exercises

11. Bernoulli Trials

---

1. Introduction

1.9. Yes, probably so. The outcomes are *correct* and *incorrect* and $p = \frac{1}{4}$.

1.10. Yes, approximately. The outcomes are *prefer A* and *do not prefer A*; p is the proportion of voters in the entire district who prefer A.

1.11. Yes, the outcomes are *red* and *black*, and $p = \frac{18}{38}$.

1.12. No, probably not. The games are almost certainly dependent, and the win probably depends on who is serving and thus is not constant from game to game.

1.15. $r(p) = p\left(2p - p^2\right)^2 + (1 - p)\left(2p^2 - p^4\right)$

1.22.

a. $k = 10$, $\mathbb{E}(Y_k) = 19.56$
b. $k = 5$, $\mathbb{E}(Y_k) = 426.22$
c. $k = 40$, $\mathbb{E}(Y_k) = 64.23$

2. The Binomial Distribution

2.29.

a. $\mathbb{P}(X = k) = \binom{20}{k}\left(\frac{1}{5}\right)^k\left(\frac{4}{5}\right)^{20-k}$ for $k \in \{0, 1, ..., 20\}$
b. $\mathbb{E}(X) = 4$
c. $\text{var}(X) = \frac{16}{5}$
d. $\mathbb{P}(X \geq 12) \approx 0.000102$. She has no hope of passing.

2.30.

a. $\mathbb{P}(Y = k) = \binom{50}{k}\left(\frac{1}{50}\right)^k\left(\frac{49}{50}\right)^{50-k}$ for $k \in \{0, 1, ..., 50\}$
b. $\mathbb{E}(X) = 1$

c. $\text{var}(X) = \frac{49}{50}$
d. $\mathbb{P}(Y \le 3) \approx 0.9822$.

2.31.

a. $\mathbb{P}(Z = k) = \binom{50}{k} \left(\frac{2}{5}\right)^k \left(\frac{3}{5}\right)^{50-k}$ for $k \in \{0, 1, ..., 50\}$
b. $\mathbb{E}(Z) = 20$
c. $\text{var}(Z) = 12$
d. $\mathbb{P}(Z < 19) \approx 0.3356$.
e. $\mathbb{P}(Z < 19) \approx 0.3330$.

2.32.

a. $\mathbb{P}(N = k) = \binom{10}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{10-k}$ for $k \in \{0, 1, ..., 10\}$
b. $\mathbb{E}(N) = \frac{5}{3}$
c. $\text{var}(N) = \frac{25}{18}$

2.33. Let Y_n denote the number of heads in the first n tosses.

$$\mathbb{P}(Y_{20} = j | Y_{100} = 30) = \frac{\binom{20}{j}\binom{80}{30-j}}{\binom{100}{30}}, \quad j \in \{0, 1, ..., 20\}$$

2.34.

a. $\mathbb{P}(Z = k) = \binom{1000}{k} \left(\frac{3}{8}\right)^k \left(\frac{5}{8}\right)^{1000-k}$ for $k \in \{0, 1, ..., 1000\}$
b. $\mathbb{E}(Z) = 375$
c. $\text{var}(Z) = \frac{1875}{8}$
d. $\mathbb{P}(Z \ge 400) \approx 0.0552$.
e. $\mathbb{P}(Z \ge 400) \approx 0.0550$.

2.36.

a. $\mathbb{P}(5 \le Y \le 10) = 0.8815$
c. $\mathbb{P}(5 \le Y \le 10) \approx 0.878$

2.37.

a. $\mathbb{P}(0.5 \le M \le 0.7) = 0.8089$
c. $\mathbb{P}(0.5 \le M \le 0.7) \approx 0.808$

2.39.

a. $\mathbb{P}(\text{at least 1 ace in 6 rolls}) = 0.6651$
b. $\mathbb{P}(\text{at least 2 aces in 12 rolls}) = 0.6187$

2.40.

a. $\mathbb{P}(\text{at least 1 ace in 4 rolls of 1 die}) = 0.5177$
b. $\mathbb{P}(\text{at least 2 aces in 24 rolls of 2 dice}) = 0.4914$

2.41. Proportion of females:

a. $m = 0.433$
b. $m_0 = 0.636$
c. $m_1 = 0.259$
d. $m_2 = 0.5$

2.42. $m_{\text{red}} = 0.168$

2.48.

a. $r_{3,2}(p) = 3\,p^2 - 2\,p^3$.
b. $r_{5,3}(p) = 10\,p^3 - 15\,p^4 + 6\,p^5$.
c. 3 out of 5 is better for $p \geq \frac{1}{2}$

2.54.

a. $b_1(p) = 1 - 2\,p$
b. $b_2(p) = 1 - 2\,p$
c. $b_3(p) = 1 - \frac{3}{2}\,p - \frac{3}{2}\,p^2 + p^3$

3. The Geometric Distribution

3.16.

a. $\mathbb{P}(U = n) = \left(\frac{5}{6}\right)^{n-1} \frac{1}{6}, \quad n \in \mathbb{N}_+$
b. $\mathbb{E}(U) = 6$
c. $\text{var}(U) = 30$
d. $\mathbb{P}(U \geq 5) = \frac{525}{1296}$

3.17.

a. $\mathbb{P}(N = n) = \left(\frac{49}{50}\right)^{n-1} \frac{1}{50}, \quad n \in \mathbb{N}_+$
b. $\mathbb{E}(Z) = 50$
c. $\text{var}(Z) = 2450$

d. $\mathbb{P}(N > 20) = 0.6676$

3.18. 0.4

3.19. Geometric with $p = \frac{18}{38}$

3.24. \$1000.

3.29. $\mathbb{P}(W = i) = \frac{2^{n-i}}{2^n - 1}, \quad i \in \{1, 2, ..., n\}$

4. The Negative Binomial Distribution

4.18.

a. $\mathbb{P}(V = n) = \binom{n-1}{2} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{n-3}, \quad n \in \{3, 4, ...\}$
b. $\mathbb{E}(V) = 18$
c. $\text{var}(V) = 90$
d. $\mathbb{P}(V \geq 20) = 0.3643$

4.19.

a. $\mathbb{P}(V_5 = m | V_{10} = 25) = \frac{\binom{m-1}{4}\binom{24-m}{4}}{\binom{24}{9}}, \quad m \in \{5, 6, ..., 20\}$
b. $\mathbb{E}(V_5 | V_{10} = 25) = \frac{25}{2}$
c. $\mathbb{E}(V_5 | V_{10} = 25) = \frac{375}{44}$

4.20.

a. $\mathbb{P}(N = n) = \binom{n-1}{2} \left(\frac{1}{50}\right)^4 \left(\frac{49}{50}\right)^{n-4}, \quad n \in \{4, 5, ...\}$
b. $\mathbb{E}(N) = 200$
c. $\text{var}(N) = 9800$
d. $\mathbb{P}(N \leq 200) = 0.5685$

4.21.

a. $\mathbb{P}(8 \leq V_5 \leq 15) = 0.7142$
c. $\mathbb{P}(8 \leq V_5 \leq 15) \approx 0.7445$

4.22. Let V denote the number of tosses needed to get 50 heads.

a. 0.0072
b. No.

4.38.

a. 0.6825.
b. 0.7102

4.44.

a. $f(k) = \binom{k-1}{3}\left(\frac{1}{2}\right)^{k-1}$ for $k \in \{4, 5, 6, 7\}$, $\mathbb{E}(N) = 5.8125$, $\text{sd}(N) = 1.0136$.
b. $f(k) = \binom{k-1}{3}\left(0.7^4\, 0.3^{k-4} + 0.3^4\, 0.7^{k-4}\right)$ for $k \in \{4, 5, 6, 7\}$, $\mathbb{E}(N) = 5.3780$, $\text{sd}(N) = 1.0497$.
c. $f(k) = \binom{k-1}{3}\left(0.9^4\, 0.1^{k-4} + 0.1^4\, 0.9^{k-4}\right)$ for $k \in \{4, 5, 6, 7\}$, $\mathbb{E}(N) = 4.4394$, $\text{sd}(N) = 0.6831$.

4.46. *A* gets \$72.56, *B* gets \$27.44

5. The Multinomial Distribution

5.10.

a. 0.0075
b. 0.0178
c. 0.205
d. 0.123

5.11. $f(u, v, w, x, y, z) = \binom{4}{u, v, w, x, y, z}\left(\frac{1}{4}\right)^{u+z}\left(\frac{1}{8}\right)^{v+w+x+y}$ for u, v, w, x, y, z nonnegative integers that sum to 4

5.13.

a. -0.625
b. -0.0386

6. The Simple Random Walk

6.10.

a. 0.7794
c. 0.7752

6.18.

a. Probability density function of M_5: $f(0) = f(1) = \frac{10}{32}$, $f(2) = f(3) = \frac{5}{32}$, $f(4) = f(5) = \frac{1}{32}$.
b. $\mathbb{E}(M_5) = \frac{11}{8}$
c. $\text{var}(M_5) = \frac{111}{64}$

6.19. $\mathbb{P}(M_{10} \le 4) = \frac{57}{64}$

6.32.

a. Probability density function of L_{10}: $f(0) = f(10) = \frac{63}{256}$, $f(2) = f(8) = \frac{35}{256}$, $f(4) = f(6) = \frac{30}{256}$,
b. $\mathbb{E}(L_{10}) = 5$
c. $\text{var}(L_{10}) = 15$

6.37. $\frac{3191}{11895} \approx 0.2683$

6.41. $\frac{3}{25}$

6.45. $f(2) = \frac{1}{2}$, $f(4) = \frac{1}{8}$, $f(6) = \frac{1}{16}$, $f(8) = \frac{5}{128}$, $f(10) = \frac{7}{512}$.

Virtual Laboratories > 11. Bernoulli Trials > Answers to Selected Exercises