

# Geometry of Fractional Brownian Sheets

Dr. Dongsheng Wu

As a class of typical anisotropic Gaussian random fields, fractional Brownian sheets arise naturally in many areas, including in stochastic partial differential equations and in studies of the symmetric Markov processes. Let  $B^H = \{B^H(t) : t \in \mathbb{R}_+^N\}$  be an  $(N, d)$ -fractional Brownian sheet with Hurst index  $\mathbf{H} = (H_1, \dots, H_N) \in (0, 1)^N$ . We prove that  $B^H$  has the property of sectorial local non-determinism. By using it as a main tool, we describe various sample path properties of  $B^H$  in terms of the Hurst index  $H$ . In particular, we determine the Hausdorff dimension of the image set  $B^H(E)$  for an arbitrary Borel set  $E \subset (0, \infty)^N$ . and we provide sufficient conditions for  $B^H(E)$  to be a Salem set or to have interior points almost surely. If time permits, we will investigate the existence and joint continuity of the local times and self-intersection local times of fractional Brownian sheets and determine the Hausdorff dimensions of the sets of  $m$ -multiple times and  $m$ -multiple points.